

Simple gas stuff

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1 Notation

k Boltzmann constant

m_p proton mass

p pressure

ρ mass density

n number density

T temperature

c_s (adiabatic) sound speed

ϵ specific thermal/internal energy

μ mean molecular weight (ex: 0.5 for ionized hydrogen)

2 Equation of state

ideal gas law

$$p = nkT = \frac{\rho kT}{\mu m_p} \quad (1)$$

gamma-law equation of state

$$p = A\rho^\gamma \quad (2)$$

adiabatic sound speed definition

$$\frac{\partial p}{\partial \rho} = c_s^2 = \gamma \frac{p}{\rho} = \frac{\gamma kT}{\mu m_p} \quad (3)$$

specific thermal energy definition (from kinetic theory)

$$\epsilon = \frac{1}{\gamma - 1} \frac{p}{\rho} = \frac{c_s^2}{\gamma} \quad (4)$$

3 Hydro conservation laws

Eulerian hydrodynamics with index notation

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial r_i} &= 0 \\
\frac{\partial(\rho u_j)}{\partial t} + \frac{\partial(\rho u_j u_i)}{\partial r_i} + \frac{\partial p}{\partial r_j} + \rho \frac{\partial \Phi}{\partial r_j} &= 0 \\
\frac{\partial(\rho e)}{\partial t} + \frac{\partial(\rho e u_i + p u_i)}{\partial r_i} + \rho u_i \frac{\partial \Phi}{\partial r_i} &= 0
\end{aligned} \tag{5}$$

Cosmological variables

$$\begin{aligned}
r_i &= a x_i \\
\frac{dr_i}{dt} &= u_i = \dot{a} x_i + a \dot{x}_i \\
a \dot{x}_i &= v_i \\
\left(\frac{\partial}{\partial t} \right) \Big|_r &= \frac{\partial}{\partial t} - \frac{\dot{a}}{a} x_i \frac{\partial}{\partial x_i} \\
\frac{\partial}{\partial r_i} &= \frac{1}{a} \frac{\partial}{\partial x_i} \\
\Phi &= \phi - \frac{1}{2} a \ddot{a} x^2 \\
\rho_p &= \rho / a^3 \\
p_p &= p / a^3
\end{aligned}$$

Now in cosmological coordinates (note I am swapping ρ and p notation here)

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \frac{1}{a} \frac{\partial(\rho v_i)}{\partial x_i} &= 0 \\
\frac{\partial(\rho v_j)}{\partial t} + \frac{1}{a} \frac{\partial(\rho v_j v_i)}{\partial x_i} + \frac{1}{a} \frac{\partial p}{\partial x_j} + \frac{\dot{a}}{a} \rho v_i + \frac{1}{a} \rho \frac{\partial \phi}{\partial x_j} &= 0 \\
\frac{\partial(\rho e)}{\partial t} + \frac{1}{a} \frac{\partial(\rho e v_i + p v_i)}{\partial x_i} + 2 \frac{\dot{a}}{a} \rho e + \frac{1}{a} \rho v_i \frac{\partial \phi}{\partial x_i} &= 0
\end{aligned} \tag{6}$$